Bayesian Methods – Guidance

Background

Leveraging information from various sources to estimate reliability using Bayesian methods is becoming more common and has many advantages. Most notably, multiple sources of prior information (e.g., operationally relevant developmental testing or operational assessments) can be incorporated; complex systems (and their structures) can be analyzed without seriously increasing the computations; and uncertainty intervals are straightforward to calculate and interpret. These techniques require thought and understanding of both the system and the statistics. In addition to the actions taken to develop a classical test plan, a Bayesian approach needs to determine a prior distribution (what information to leverage) and establish an analysis framework (how to incorporate prior information).

Data Quality

Relevant prior information to be incorporated in OT analysis may include previous developmental or operational test data, engineering analyses, or information from modeling and simulation. The origin and quality of prior information should be indicated in the TEMP.

Any type of prior test data is reasonable to use in prior construction. However, the relevance of the prior information to the current data (i.e., DT data for OT analysis) should impact how heavily the prior information is weighted. The main point is not to introduce a bias into the current analysis if past test data do not support the results of the current test data.

Incorporating Prior Information

The prior data should always be used to aid in choosing which areas within the operational envelope to include in testing: knowing what combination of settings could be difficult or easy for the system, etc. Another approach to use the prior information is to shorten the test (or accepting a less powerful test) with the knowledge that the prior information will serve as the basis for the analysis. Assurance testing is a formal procedure to plan a test which combines information from various sources to reduce the amount of testing required to meet a requirement. See the references below for more information. For some systems, it might be reasonable to plan the OT to focus on the additional operational effects (i.e. the addition of an operator), because the prior information gives a good idea of the performance of the base system. Extremely informative priors which allow virtually no learning from OT data must be avoided.

Analysis Plan

The TEMP must indicate the intended analysis plan for the OT data when a Bayesian approach is proposed. That analysis plan should establish how the prior information is to be included in the operational evaluation.. For example, in a reliability analysis using Bayesian methodologies the operational test evaluation would specify the distribution that will be used to analyze the data (e.g., Exponential or Weibull), the prior distribution , and how it was derived from existing data.

General Guidelines for when Bayesian Methods are Appropriate

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There are a few indications that Bayesian methods will be applicable and worth the additional effort:

- When relevant and defensible prior information is available. The Bayesian example illustrates a case when extensive information from DT is available at the subsystem level and the operational testing focuses on the integration of these subsystems with an operational user. Even when including prior information, the prior must have enough variability to allow the estimates to move away from what was previously seen if the data support such values.
- When assessing system or kill-chain reliability. Such analyses generally involve combining information from many areas or subsystems under a possibly complex system structure. Obtaining interval estimates for any system model is straightforward in a Bayesian analysis, but not in traditional analyses. Moreover, if any subsystem or component in a kill-chain or system analysis has zero failures, point and interval estimates are still attainable.
- To avoid unrealistic point estimates and to obtain interval estimates when measuring mean time between failure for short tests or highly reliable systems and zero failures are expected.

Bayesian Principles

Some basic, overarching principles to consider when planning a Bayesian analysis:

- Start with the properties of the parameter of interest: if a parameter needs to be positive, choose a distribution that is also non-negative.
- Decide on what prior information to use and how relevant it is to operational test evaluation.
- Allow for the analysis to change freely based on the data observed in operational testing.
- Check impact of your prior assumptions: explore prior predictions for bias and re-check in the analysis with a sensitivity study. A good model should be fairly robust to prior specifications.

Bayesian Example

References

Hamada, M.S., Wilson, A.G., Weaver, B.P., Griffiths, R.W., and Martz, H.F. (2014) "Bayesian Binomial Assurance Tests for System Reliability Using Component Data," *Journal of Quality Technology*, 46, 24-32.

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