Bayesian Methods – Example

System Description

A new mobile lab system is intended to analyze environmental samples for the presence of chemical, biological, and radiological material, and report the analytical results to directly support commander's force protection and force health surveillance decisions. Each subsystem (chemical, biological, and radiological) is comprised of a collection of components of various sensitivity, speed, and cost to run. Each set/system will be tailored to the specific operational user and their mission needs by incorporating specific capabilities from a common suite of Commercial-Off-the-Shelf (COTS) and Government-Off-the-Shelf (GOTS) analytical technologies and components. KPP performance requirement for each subsystem is to detect 85 percent of samples that come into the lab.

Prior Information

The subsystem components have completed multiple phases of testing to determine detection performance curves. Each phase increases the operational relevance of the testing: Phase 1 tested various targets at different concentrations in a pristine matrix on each component and Phase 2 tested various targets at different concentrations in operationally representative matrices such as soil, food, or swabs. There have been approximately 5500 tests on the subsystem to characterize the detection performance of the components for the target matrix combinations. Prior information to be incorporated in the analysis comes exclusively from prior test data that will be down-weighted for OT to take into account departures from operational realism (i.e. lab technicians versus soldier operators).

A logistic regression was used to analyze the Phase 2 data for with the factors target, matrix, and concentration. Dispersed priors are placed on each regression coefficient to obtain performance curves for each of the components and the target/matrix combinations:

$$\boldsymbol{logit}(\boldsymbol{P_D}) = \boldsymbol{\beta_1} * \boldsymbol{conc} + \boldsymbol{\beta_{2,matrix}} + \boldsymbol{\beta_{3,target}}$$

$$(\beta_1, \boldsymbol{\beta_2}, \boldsymbol{\beta_3}) \sim \mathit{Multivariate\ Normal}(\mathbf{0}, \mathbf{10^3}\boldsymbol{I})$$

Here, the evaluation explicitly forced a dependence on concentration (generating a curve) while leveraging all device runs to learn about each target/matrix combination[1]. The Figure 1 shows an example performance curves from the regression analysis.

Scoping the test

For the OT phase, each subsystem will be tested with various targets in various matrices by an operator according to sample processing and triage procedures. Most samples will be tested on multiple components within a subsystem, and then a final call will be made by the operator.

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1 MCMC techniques were used to generate posterior distributions for the regression coefficients. These posterior distributions can be used to calculate posterior distributions of the performance curves shown in the figure across concentration for any target/matrix combination from all four devices.

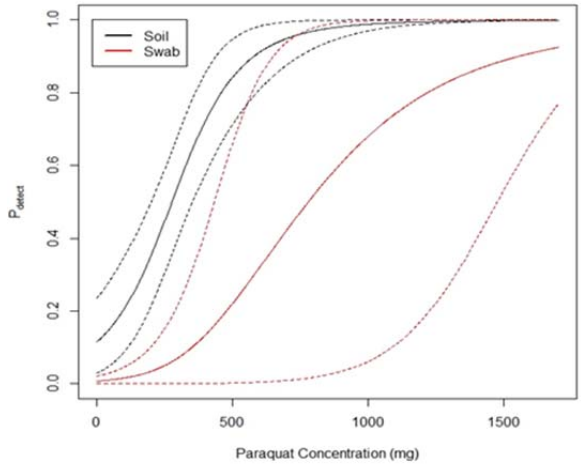

Figure 1. Probability of Detection for Paraquat Concentration in Soil and Swab

In the case of the chemical subsystem, the posterior from the Phase 2 analysis with an added degradation factor for moving from DT to OT serves as the basis for the assurance testing algorithm (see Hamada et.al. 2008).The OT plan needs to have 6 different concentration levels of 20 target/matrix combinations. The combinations are selected randomly from a list of threat representative agents of interest to the users as illustrated in Table 1.

Table 1: Target and matrix for OT

Target	Matrix
COI2 impure	Sand
GF	Soil
Sulfuric Acid	Pristine
VX	Sand
Methanol	Swab
GB-WGA	Air
COI3 pure	Vegetation
Lewisite	Sand
Sodium Cyanide	Water
GD	Soil
2-chlorovinylarsonic acid	Swab
Formaldehyde	Water
Paraquat	Vegetation
Octamethylpyrophosphoramide	Vegetation
Allyl Alcohol	Swab
Ammonia	Water
Thiodiglycol	Swab
Pinacolyl Methylphosphonic Acid	Sand
CVAOA	Soil
Methyl Bromide	Air

Determining Concentration Levels

Where information about threat representative or toxicity concentration levels is known, the OT concentration will be set at these levels. However, this information for some target/matrix combinations may not be known. The Phase 1 and Phase 2 analysis provides some insight into the range of values that each component, and each subsystem, can or might have difficulty detecting.

The lowest concentration of a given target/matrix combination will be set at the most sensitive device in the subsystem's $P_{detect} = 0.5$. This means that the lowest concentration level of any sample provided to the subsystem is set where the most sensitive of the components has a 50 percent chance of detecting. As shown in Figure 2 below, Component 2 of the biological subsystem can detect SEB in smaller concentrations than Component 1. The smallest of the concentrations in each matrix will be set where the performance curves for Component 2 cross 0.50. This would be a sample that the subsystem would have difficulty detecting, but controls the risk of setting all concentrations out of range for any component to detect.

Some concentration levels might be set so as to decrease the width of the performance curve to date. For instance, the analysis shown in Figure 1 suggests that 2 concentrations for Paraquat on a swab could be added between 500 and 1000 mg and at least one above 1000 mg on Component 1 of the chemical subsystem to add more information where the intervals are widest. By combining threat or toxicity level intelligence information and the Phase 1 and Phase 2 analysis, the 6 concentration levels for each agent/matrix combination can be set.

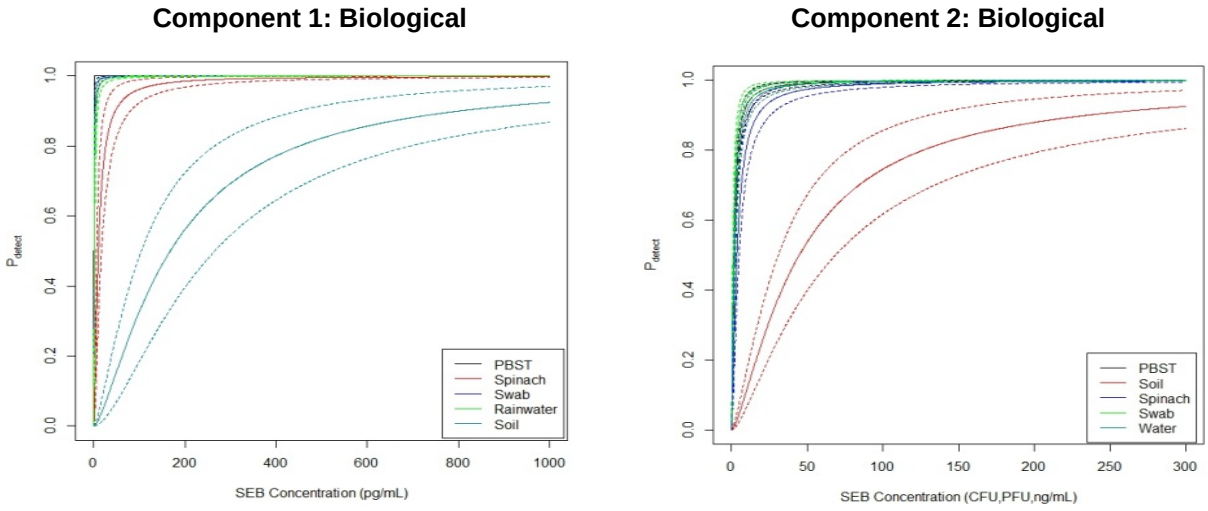

Figure 2. Probability of Detection for SEB Concentration for Two Components

Analysis Plan

To analyze the OT data, a logistic regression will again be used for each component of each subsystem with target, matrix, and concentration as factors. The Phase 2 posterior distributions will be used for the prior of the OT regression coefficients, with some additional variability.

$$\boldsymbol{logit}(\boldsymbol{P_D}) = \boldsymbol{\beta_1} * \boldsymbol{conc} + \boldsymbol{\beta_{2,matrix}} + \boldsymbol{\beta_{3,agent}}$$

$$(\beta_1, \boldsymbol{\beta_2}, \boldsymbol{\beta_3}) \sim Multivariate\ Normal(\boldsymbol{\mu_{Phase\ 2}}, \boldsymbol{c\Sigma_{Phase\ 2}})$$

The probability of a subsystem failing to detect is a function of how the components of the subsystem are structured. There are many types of system structures; some simple and commonly used are in series (all of the components must detect), in parallel (at least one component must detect), and k-of-n (at least k of the n components must detect). Here, a k-of-n system structure will be used based on the CONOPS. That is, the overall laboratory identification for a sample is made if at least 2 components in the subsystem detect a target in that sample. This will incorporate the component level information as well as account for the operator call.